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TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

NACA-TM No. 395

REMOVING BOUNDARY LAYER BY SUCTION

By J. Ackeret

From "Zeitschrift des Vereines deutscher Ingenieur"
August 28, 1926

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US Department of Commerce
Springfield, VA. 22151

Washington
January, 1926

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL MEMORANDUM NO. 395.

REMOVING BOUNDARY LAYER BY SUCTION.*

By J. Ackeret.

Through the utilization of the "Magnus effect" on the Flettner rotor ship, the attention of the public has been directed to the underlying physical principle. It has been found that the Prandtl boundary-layer theory furnishes a satisfactory explanation of the observed phenomena.

Prandtl, in his fundamental treatise,** describes an experiment intended to serve as the basis for the conception that the vortex phenomena (with whose existence the great forces of resistance, at first theoretically incomprehensible, are so closely connected), are produced by the separation phenomena in the boundary layer. The present article deals with the prevention of this separation or detachment of the flow by drawing the boundary layer into the inside of the body through a slot or slots in its surface.

This method gives results which extend far beyond the purely physical domain and are of practical importance for obtaining certain technical information on the subject of flow. The following experiments demonstrate the possibility of producing,

* "Grenzschichtabsaugung." From "Zeitschrift des Vereines deutscher Ingenieur," August 28, 1928, pp. 1153-1158. See also N.A.C.A. Technical Memorandum Nos. 374 and 388.

** L. Prandtl, "Verhandlungen des dritten internationalen mathematischen Kongresses" in Heidelberg, 1904, p.484. Published in Leipzig in 1905.

through suction, certain forms of flow which stand in surprising disagreement with the so-called "hydraulic sense."

The technician has long been familiar with the fact that the retardation of the velocity in flowing fluids (conversion of kinetic energy into energy of pressure) is a far more difficult task than the acceleration of the flow. The loss of energy in nozzles where pressure is converted into velocity is sometimes less than 1%. A suction tube capable of converting velocity into pressure with 99% efficiency has never been discovered and probably never will be. An efficiency of 85% is considered very good.

This fundamental difference is obviously due to the fact that the effects of viscosity differ greatly in pressure increase and decrease. The theory of nonviscous flow has been developed so far as to leave only mathematical difficulties yet to be overcome. Physical difficulties no longer exist. It is now very natural to think that nonviscous or frictionless flow is the limit toward which a viscous flow tends with constantly diminishing viscosity coefficients. Experience has shown, however, that this is not always the case. Even when the inertia forces greatly exceed the viscosity forces (when the so-called "Reynolds Number" is large), the observed relative energy losses are of almost constant magnitude.

This difficulty disappears, however, when a property of fluids, adhesion to solid surfaces, is considered, which, unfortu-

nately, is always regarded as included in the viscosity. While the viscosity can be diminished as much as desired (by heating, etc.), the adhesion is a condition, which is, so to speak, always 100% present. Enough emphasis can not be placed on the importance of this fact. The molecular forces which bind the fluid molecules together suffice, in conjunction with very small viscosity forces, to account for the extraordinary deviations of the actual flows from the logical results of the nonviscous theory.

I have already mentioned that the inertia forces are generally much greater than the viscosity forces. Disregarding the flow of very viscous fluids (lubricating oils, etc.), the preponderance of the inertia forces is especially great in the technically most important fluids. In order to illustrate this, we will consider a simple example. We will assume that water is flowing at the rate of 2 m (6.56 ft.) per second against a round pile of 2 meters diameter. A water molecule at the position A (Fig. 1) 1 meter (3.28 feet) in front of the pile is retarded, since it is moving directly toward the center of dynamic pressure P. The viscosity forces also participate somewhat in this retardation, but it can be easily calculated that they cause only a millionth part of the total retardation. Therefore the viscosity forces in the fluid can not come into consideration at first.

The adhesion of the fluid to the fixed wall, however, en-

ables the viscosity forces to take effect. The viscosity tensions are obtained as the product of a viscosity factor and the velocity decrease across the direction of flow. When, however, the fluid near the wall flows swiftly over the exceedingly thin layer of adhering molecules, the second factor (the velocity decrease) is almost infinitely great, so that, even with a small viscosity factor, the product may attain a non-negligible magnitude.

In fact, the following can be observed. A thin layer of retarded molecules is formed on the surface of the obstructing body, the thickness of this layer gradually increasing in the direction of flow. The question regarding the effect of the viscosity now reverts to the effect of this boundary layer. The most important point would now be a consideration of the mechanics of this boundary layer. Since, however, this subject has already been exhaustively treated in rotor literature (including this magazine), I will here give only a brief summary.*

Roughly speaking, the boundary layer is subjected to three forces: the retarding viscosity forces W , proceeding from the wall; the accelerating viscosity forces S , from the outer more swiftly flowing fluid; and the pressure forces D exerted from without. In pipes and diffusers, for example, these are chiefly

determined by the cross-sectional relations. It all depends on

* A. Betz, "Zeitschrift des Vereines deutscher Ingenieure," Jan. 3, 1925, pp. 9-14 (N.A.C.A. Technical Memorandum No. 310). L. Prandtl, "Naturwissenschaften," 1925, pp. 93-108 (N.A.C.A. Technical Memorandum No. 367). J. Ackeret, "Das Rotorschiff," Göttingen, 1925.

the sum $D + S + W = K$. In outflow terminals, D is positive (pressure decrease), S is always positive, W is always negative and generally of greater magnitude than S . K is here positive, that is, the boundary layer is affected more by the accelerating forces. In suction pipes, however, the negative D may produce a negative K , which means a retardation of the boundary layer particles and consequent danger of separation. The separation is occasioned by the retardation of the fluid in the boundary layer till its motion is stopped and a return flow sets in, which causes a turbulent accumulation and forces the flow away from the wall. It has been shown by Prandtl, Blasius, Karman, Pohlhausen and others how these qualitative results can be more accurately determined. Although it is not yet possible to follow, with mathematical accuracy, the formation of the boundary layer with accelerated and retarded motion, an early solution of this very important problem is hoped for, in view of the progress already made in turbulence researches.

The separation of the flow from the fixed walls has very far-reaching results. In the first place, the desired flow is not obtained and, in the second place, we obtain, in and behind the separation zone, a very turbulent flow, which is designated in German as "Totwasser" (dead water) and in which the energy of flow is constantly being converted into heat.

Hitherto the separation has been prevented by suitably shaping the walls. Diffusers are slightly widened into the form of

cones. Airship hulls, airplane fuselages, wings, etc., are "streamlined." The purpose of these measures is to give the zone of pressure increase such a shape that the pressure increase per unit length will be kept as small as possible and that the accompanying force S can act on large areas. The total pressure increase must, of course, be restricted.

The Magnus effect provides another possibility for preventing the separation, the viscosity force W being removed by the accompanying motion of the surface. If we could make the surface move so that it would have at every point the velocity of the theoretical flow, no boundary layer could then be developed. Structural difficulties, however, generally prevent the application of this principle.

There is still a third possibility, the already-mentioned removal of the boundary layer by suction. Its operation can best be understood from an example. As such we will choose a strongly divergent diffuser or exit cone (Figs. 2-4). Without further measures the flow soon separates, since the boundary layer can not proceed against the strong pressure increase. The boundary layer increases greatly in thickness before separating. If now an opening is made at the point A (Fig. 2), where the thickening begins, and the boundary-layer material is drawn off through the wall, the separation can no longer occur, because the retarded matter, which would displace the normal flow, has disappeared. The flow then conforms at this point

(Fig. 3), but a new boundary layer begins to develop which, under some circumstances, with further pressure increase, may again be in danger of separating, thus necessitating further openings. The removal by suction can be made continuous by making the whole wall of perforated sheet metal (Fig. 4). Further on we will see that this arrangement is favorable for certain purposes.

For the practical application there are two especially important questions: 1. How about the stability of the flow thus produced? Can it be permanently maintained? 2. How much power is required for the suction?

In reply to question 1, it may be said that difficulties can arise, when the openings are very far apart or when, for the sake of saving power, they are introduced as nearly as possible to the point of immediate danger of separation. If, for instance, through some accidental disturbance, the flow separates before reaching the suction holes, it can again conform, only when all the turbulent fluid is drawn off with sufficient rapidity. In a much disturbed flow, it is therefore preferable to apply the suction a little in advance of the point where there is any danger of separation. Perforated sheet metal is suitable for this purpose.

The second question is very important, since the economy of the whole process depends on its answer. The power of the suction pump is found from the suction pressure and the amount of

fluid removed. The amount can be roughly estimated from the boundary-layer calculations. We obtain mathematically very small quantities, whereby it is to be borne in mind that the boundary layer in a large diffuser is relatively thinner than in a small one. The walls in contact with the stream should be as smooth as possible, since roughness increases the thickness of the boundary layer. Likewise all projecting edges should be avoided, since they cause local separation and the turbulent accumulations thus produced must then also be removed by suction. The suction pressure, to be introduced into the power calculations, is chiefly determined by the pressure at the suction point, but there are additions to be made for the pressure losses in the suction openings, which can be quite large under certain conditions. When there are several suction stages, it is better to employ different suction pressures. No special suction pump is necessary, where the exhaust fluid can be simply led into a region of lower pressure. The situation is analogous to that of multi-stage pumps or turbocompressors, where the boundary-layer material can be led back to a previous stage. There are here many structural possibilities.

On the basis of certain experiments performed in the Göttingen aerodynamic laboratory, we will now show how the removal of the boundary layer by suction opens up some entirely new flow possibilities.*

* I would not longer neglect to mention that Dr. A. Betz has taken the liveliest interest in all the researches related to the removal of the boundary layer and has constantly assisted me by his friendly counsel.

1. Sphere from which the boundary layer is removed by suction (N.A.C.A. Technical Memorandum No. 388).— The sphere is usually taken as the example to show how poorly the theory of the potential flow agrees with the reality. According to this theory the flow before and behind the sphere should be perfectly symmetrical and consequently develop no resistance. We know, however, that this is not even approximately true. Behind the sphere there is an extensive region of turbulence, as shown in Fig. 5, through the introduction of smoke, the air flow being from the right. When the suction is applied, the flow conforms to the surface of the sphere far back, the turbulent region becomes smaller and might ultimately be entirely eliminated by a better arrangement and stronger suction (Fig. 6). The removal by suction is accomplished in three stages, through annular strips of wire gauze. These strips are likewise shown in Fig. 7 (drawn from Fig. 6). It is seen that at a the separation occurs some distance behind the last strip, while at b, it sets in immediately. That the flow at a is not permanently stable, however, is shown by the fact that turbulence is developed at c. This turbulence can only come from a flow similar to b which prevailed a few instants earlier. The resistance of the sphere decreases simultaneously with the application of the suction and approximates that of well-shaped airship hulls. These experiments will be more minutely described in an article soon to be published in "Zeitschrift für Flugtechnik und Motor-

luftschiffahrt."

2. Airfoils.— For airfoils the removal of the boundary layer by suction is prospectively of special importance. The maximum lift, which plays such a decisive role in the landing of airplanes, is limited by the separation of the flow on the suction side of the wings at large angles of attack. Fig. 8 shows an airfoil with the boundary layer removed by suction from the upper side. The angle of attack is not less than 45° , but the flow nevertheless conforms smoothly to the surface of the airfoil, as indicated by the position of the threads in the air stream. The maximum lift reaches threefold that of an ordinary airfoil. More detailed information concerning this experiment is given in No. 4 (November, 1925) of "Vorläufige Mitteilungen der Aerodynamischen Versuchsanstalt zu Göttingen" (N.A.C.A. Technical Memorandum No. 374).

3. Deflection of a free air flow 180° by a unilateral removal of the boundary layer by suction.— Figs. 9-12 show that very singular forms of flow can be produced by removing the boundary layer by suction. A free air flow was made to change its course 180° around a semicylindrical surface from which the boundary layer was removed by suction. When the suction was removed, the flow went straight ahead without the least regard to the semicylindrical surface. In Figs. 9-10, which are further elucidated by the diagrammatic Figs. 11-12, the air flow is indicated by the silk threads. While the flow is straight ahead in

Fig. 9, its deflection is very clearly shown in Fig. 10. On the side where the flow is free, there is a strong unavoidable mixing with the surrounding quiet air. This is shown by the fluttering of the upper threads. Of course it is also possible to prevent the separation from any kind of curved surfaces and corresponding experiments have actually shown that the flow losses in the deflection can be diminished.

4. Diffuser from which the boundary layer is removed by suction.— In a final example, we will consider the experimental details somewhat more thoroughly. This example concerns the flow in strongly diverging diffusers with circular cross sections. Figs. 13-14 show the forms tested and the hatched surfaces indicate the actual diffusers or exit cones, the points where the pressures were measured being indicated by Roman numerals. The width of the single suction slot is indicated by s . A current rectifier was placed in the entrance cone, in order to remove the disturbing rotation of the air caused by the propeller. The exit cone was made very long, since it was found that pressure increase could be observed a long way from the narrowest cross section, due to the intermingling of portions of the air stream flowing at different velocities.

The mouth of the exit cone could be throttled, thus rendering it possible to vary the pressure at the suction point, so that the boundary layer would automatically flow through the slot and the pump could be dispensed with. In both cases the

ratio of divergence is 4. The pressures were measured with mercury columns and plotted in Figs. 15-16 in percentages of the maximum pressure increase computed from the kinetic energy in the constricted section (corresponding to a velocity head h_0) and from the chosen cross-sectional ratio. The abscissa represents the ratio of its distance x (from the smallest cross section) to its diameter d . The elimination of the suction is indicated by $s = 0$. It is evident from the pressure data that the efficiency of very divergent diffusers can be improved by suction, though it is of little use to provide suction for good tapering forms. Tables I and II contain data for the types A and B at certain "throttle" positions.

Table I - Type A.

s	h_0	Q	Q_a	$\epsilon = \frac{Q_a}{Q}$	$h_{IV}-h_{II}$ theo- retical	$h_{IV}-h_{II}$ meas- ured	h_a	η	α
mm	m water	l/s	l/s	%	m water	m water	m water	%	%
0	8.42	1.93	0	0	7.93	5.94	--	75	0
0.1	8.50	1.94	0.018	0.9	8.00	6.25	4.70	77.8	0.67
0.2	8.32	1.92	0.031	1.6	7.80	6.50	4.83	82.3	1.25
0.3	8.48	1.94	0.040	2.1	7.95	6.65	4.66	82.3	1.55
0.4	8.41	1.93	0.056	2.9	7.90	6.65	4.70	82.4	2.15

Table II - Type B.

s	h_o	Q	Q_a	$\epsilon = \frac{Q_a}{Q}$	$h_{IV} - h_{II}$ theoretical	$h_{IV} - h_{II}$ measured	h_a	η	α
mm	m water	l/s	l/s	%	m water	m water	m water	%	%
0	8.45	1.94	0	0	7.92	3.96	--	50	0
0.2	8.30	1.91	0.034	1.8	7.80	4.78	3.89	61	1.1
0.3	8.47	1.94	0.046	2.4	7.93	5.09	4.17	63	1.55
0.4	8.45	1.93	0.07	3.65	7.92	5.88	4.35	72	2.5
0.5	8.81	1.97	0.09	4.50	8.25	6.67	4.58	78	3.1
0.6	8.46	1.94	0.094	4.9	7.92	6.68	4.45	81.2	3.4
0.7	8.69	1.96	0.15	7.7	8.12	6.80	4.46	79.5	5.3

The quantity of water Q is calculated from the pressure drop $h_I - h_{II}$ with consideration of the inflow velocity in section I and under the assumption of a nozzle factor of 97.5%, in order to allow in some measure for the formation of a boundary layer on the walls. The quantity Q_a removed by suction was determined directly by means of a measuring vessel and recorded in column 5 in % of the total quantity of water ($\epsilon = \frac{Q_a}{Q}$). The theoretical pressure increase does not equal the total velocity head h_o , which the water has in the narrowest cross section, since it is not retarded to zero velocity, but only to a fourth of the velocity in the narrowest cross section.* Hence there remains an excess velocity head of $\left(\frac{1}{4}\right) h_o = \frac{1}{16} h_o$ and the theoretical pressure increase $h_{IV} - h_{II}$ is $\frac{15}{16} h_o$. The pressure increase $h_{IV} - h_{II}$ is given in column 7.

* With suction, however, this is not strictly accurate, since the quantity of fluid flowing through the enlarged cross section is diminished by the quantity removed by suction. The calculated pressure increase is therefore a little too large (less than 1%).

The suction effect is determined as follows. We proceed from the assumption that all the water, including that removed by suction, is to be brought ultimately to the pressure head h_{IV} . The latter, however, is greater than the pressure outside the suction slot. A pump is therefore necessary to raise the boundary-layer material from the surrounding pressure to h_{IV} . The pressure head, which the pump must overcome, is given in column 8 as the suction pressure h_a . I have put the efficiency of the pump at 75%, at which, under favorable conditions, the efficiency of the actuating electric motor can also be maintained. The efficiency η was calculated by the formula

$$\eta = \frac{Q (h_{IV} - h_{II})}{\frac{15}{16} Q h_o + \frac{Q_a h_a}{0.75}} = \frac{h_{IV} - h_{II}}{\frac{15}{16} h_o + \frac{\epsilon h_a}{0.75}}.$$

The values thus found are given in column 9.

Lastly, column 10 shows what fraction (α) of the power passing through the narrowest cross section of the suction pipe must be utilized as the suction power (with 75% pump efficiency). The values obtained enable us to hope for a favorable result in practice. I will indicate briefly the dimensions required for the suction pump in a given case.

A low-pressure water turbine of about 10,000 HP. at 7 m (23 ft.) head and 126 m³ (4450 cu.ft.) per second is provided with a suction pipe according to Table II, corresponding to $s = 0.6$. The diameter of the rotor wheel is about 5 m (16.4 ft.).

The kinetic energy of the water in the narrowest cross section is then about 28.5% of the total energy, i.e., a value such as occurs in recent power plants. The suction slot would have to be about 1.5 m (4.9 ft.) below the rotor and have a width of 22 cm (8.66 in.). In connection with the latter number, it should be noted that a proportional enlargement of the dimensions of the relatively small experimental model is a too unfavorable assumption. The boundary layer can safely be assumed to be relatively thinner in the full-sized apparatus and the size of the slot and the quantity of fluid removed by suction can be correspondingly smaller. If, however, we retain the model value in spite of this fact, we remove 6.2 m^3 (219 cu.ft.) per second, generate a suction pressure of 1.05 m (41.3 in.) and a suction (at 75% pump efficiency) of 116 HP., which is not much more than 1% of the power of the turbine. With continuous suction, it may be possible to obtain still better results. Further experiments are being planned.

The removal of the boundary layer counts under certain circumstances, even in problems of heat transmission. This is evidenced by the fact that the portions of radiator pipes first brought into contact with the flow, and where the boundary layer is still thin, transmit the heat considerably better than the following portions. For this reason automobile and airplane radiators are made short. The removal of the boundary layer at a few points, however, extends the favorable conditions of the

first portion, with correspondingly better heat transmission. In wing radiators, as recently used on airplanes, where the boundary layer is especially thick, due to the pressure increase on the upper or suction side of the wing, the heat transmission would probably be greatly improved by removing the boundary layer by suction.

The results, as a whole, raise the hope that we may some time succeed in relieving engineers, through the removal of the boundary layer by suction, of the necessity of employing very special shapes. Then we shall be able to put to practical use much of the large amount of technical information we have acquired in frictionless or nonviscous hydrodynamics. We must, however, first obtain further theoretical and experimental bases.

A p p e n d i x

On the Theory of Removing Boundary Layer by Suction.*

It seems possible to establish a theory for the case of constant removal of the boundary layer by suction and for moderate pressure increases on the basis of the so-called Prandtl-Karman "SiebteI" Law.**

We will next consider the behavior of the boundary layer on

* Dr. Tollmien, Göttingen, some time ago developed a theory of removal by suction on the basis of laminar boundary-layer flow, on which he is still to report. I have here discussed the practically more important turbulent flow.

** Von Karman, "Zeitschrift für angewandte Mathematik und Mechanik," Vol. I (1921), p. 233.

a flat plate without suction (Fig. 17), as represented by

$$u = U \left(\frac{y}{\delta} \right)^{1/7} \quad (1)$$

in which U is the uniform velocity outside the boundary layer, δ the thickness of the boundary layer and y the distance from the wall. Moreover, the shearing stress on the wall is

$$\tau = 0.0225 \rho \frac{\nu^{1/4} U^{7/4}}{\delta^{1/4}} \quad (2)$$

in which ρ = density of fluid and ν the kinetic viscosity. It is known that the thickness of the boundary layer increases in the direction of the flow, since new portions of the originally undisturbed fluid are constantly retarded. The thickness δ is increased, in the distance dx , by

$$d\delta = \frac{d\delta}{dx} dx.$$

If the boundary layer (in harmony with what has been observed) has a velocity distribution according to equation (1), it must include, from the undisturbed fluid, a certain quantity q per unit area and time. The value of q , for the flow along a plate, is found as follows:

$$q b dx = b dx \frac{d}{dx} \int_0^{\delta} u dy,$$

which, according to equation (1), is equivalent to

$$\frac{7}{8} b dx U \frac{d\delta}{dx}.$$

Hence

$$q = \frac{7}{8} U \frac{d\delta}{dx}. \quad (3)$$

in which b is the width of the surface considered. Since, moreover, this quantity occurs with the undisturbed velocity U , it brings the momentum

$$\rho q U = \frac{7}{8} \rho U^2 \frac{d\delta}{dx} \quad (3a)$$

With the aid of the momentum theory, it is now easy to compute the increase $d\delta/dx$. It is, namely,

$$b dx \rho q U = \tau b dx + b dx \frac{d}{dx} \int_0^\delta \rho u^2 dy$$

or

$$\rho q U = \tau + \frac{7}{9} \rho U^2 \frac{d\delta}{dx}$$

the solution of which gives

$$\frac{d\delta}{dx} = \frac{72}{7} \frac{\tau}{\rho U^2} = 0.231 \frac{v^{1/4}}{U^{1/4} \delta^{1/4}} \quad (4)$$

From (3a) and (4) we obtain further

$$\rho q U = 9 \tau \quad (5).$$

We now come to our real task. There is supposed to be a retarding pressure-increase $\frac{dp}{dx}$. Furthermore, a portion of the boundary layer is supposed to be drawn into the inside of the body. It is not possible to make a definite statement on the amount to be removed by suction. The task, however, becomes perfectly clear, if we make the following assumptions:

1. The velocity distribution shall be given by formula (1) afterwards as well as before.

2. The shearing stress shall likewise be determined by formula (2).

3. The momentum entering from without shall even now have the value given in formula (5).

4. $\frac{d\delta}{dx}$ shall be calculated with the momentum law with consideration of the pressure increase. There then arises, with the continuity equation within the boundary layer, a disagreement, which can be eliminated only by removing by suction a certain quantity q_1 (at zero velocity).

If we make allowance for the decrease in the outside velocity U with pressure increase, the momentum theory reads

$$b \, dx \, \rho \, q \, U = b \, dx \, \frac{d}{dx} \int_0^\delta \rho \, u^2 \, dy + b \, dx \, \tau + b \, dx \, \frac{dp}{dx} \delta$$

or

$$\rho \, q \, U = \rho \, \frac{7}{9} \frac{d}{dx} (U^2 \delta) + \tau + \frac{dp}{dx} \delta.$$

According to assumption 3, the left side equals 9τ , introducing which we obtain

$$8\tau = \left(\frac{7}{9} 2 \rho U \frac{dU}{dx} \delta + \rho U^2 \frac{d\delta}{dx} \right) + \frac{dp}{dx} \delta.$$

The outside flow then conforms to Bernouilli's law

$$\rho U \frac{dU}{dx} = - \frac{dp}{dx} \quad (6)$$

and hence

$$8 \tau = - \frac{5}{9} \frac{dp}{dx} \delta + \frac{7}{9} \rho U^2 \frac{d\delta}{dx}.$$

Then

$$\underline{\frac{d\delta}{dx} = \frac{72}{7} \frac{\tau}{\rho U^2} + \frac{5}{7} \frac{dp}{dx} \frac{\delta}{\rho U^2}} \quad (7)$$

We now come to the application of the continuity equation to the boundary-layer flow. The quantity flowing in through AB equals $\frac{9\tau}{\rho U} b dx$ according to assumption 3. A greater quantity generally flows out through BC than through AD. The excess is

$$b dx \frac{d}{dx} \int_0^\delta u dy = \frac{7}{8} b dx \frac{d}{dx} (U\delta).$$

Reduced to unit area:

$$\frac{7}{8} \left(\frac{dU}{dx} \delta + U \frac{d\delta}{dx} \right).$$

With the aid of equations (6) and (7), it is further transformed to

$$\frac{7}{8} \left(- \frac{dp}{dx} \frac{\delta}{\rho U} + \frac{72}{7} \frac{\tau}{\rho U} + \frac{5}{7} \frac{dp}{dx} \frac{\delta}{\rho U} \right).$$

Referred to unit area and time, the difference between the total inflowing and outflowing quantities is

$$\frac{9\tau}{\rho U} - \frac{9\tau}{\rho U} + \frac{1}{4} \frac{dp}{dx} \frac{\delta}{\rho U}$$

hence a value differing from zero and, in the sense of assumption 4, we find, for the difference to be removed by suction,

$$q_1 = \frac{1}{4} \frac{dp}{dx} \frac{\delta}{\rho U} \quad (8)$$

For any given flat flow according to equation (7) at any initial value of δ , there is generally no difficulty in determining the whole course of the boundary-layer thickness, by calculating U and $\frac{dp}{dx}$ according to the ordinary method of the potential theory. The quantity to be removed by suction is relatively very small, so that the flow deviates but slightly from the potential flow. The quantity can then be calculated according to equation (8). Furthermore, the transmission to rotational hollow spaces is easily possible.

It is still rather uncertain as to whether our assumptions are correct. We can, however, safely assert that the calculation yields correct results for small pressure increases, because, for vanishing $\frac{dp}{dx}$, equation (7) is convertible into the well-established equation (4), while, according to equation (8), q_1 naturally vanishes. With vanishing pressure increase, no removal by suction is necessary. With greater increases, it is to be expected that the assumptions will not all be strictly correct. Assumption 3, in particular, will have to be modified, since the entering momentum will then probably be greater.*

Are we then certain that there is no danger of separation? From equation (7) we find that δ increases very gradually. It can not therefore become infinite (separation), except in the

* I obtained this viewpoint from a discussion with Betz, who kindly assisted me in regard to assumption 3. See also Prandtl, "Zeitschrift für angewandte Mathematik und Mechanik," 1925, p. 136.

vicinity of the center of dynamic pressure. Likewise no return flow can occur, since assumption 2 presupposes a finite positive shearing stress τ . Although no real stability investigation has been made, we need not, for these reasons, entertain any doubt of the stability. We will not here undertake the computation of special examples. I hope to be able to recur to this subject, after further experiments have been tried. The purpose of the above discussion is only to give a preliminary survey of the theoretically anticipated quantities to be removed by suction.

S u p p l e m e n t

The three accompanying pictures were shown in connection with my Zurich lecture. Water, whose motion could be rendered visible by strewing powdered aluminum on it, flowed through an open horizontal rectangular trough. This trough was greatly constricted by placing in it two bent metal sheets. The stream, being forced to pass through this constriction, was thereby greatly accelerated. Had the potential flow prevailed after passing through the constriction, it would have become slower and uniformly filled the whole cross section. It did not do this, however, but continued as a free stream with its acquired velocity (Fig. 18), having separated from both walls. When the boundary layer was removed by suction through the slots in one of the walls, the flow followed that wall without separation

(Fig. 19). Lastly, when the suction was applied symmetrically to both walls, it was found possible to eliminate all the turbulence and produce an approximately potential flow (Fig. 20).

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

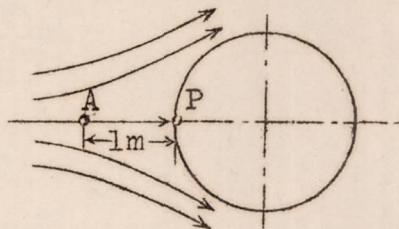


Fig.1 Flow in front of the center of pressure.

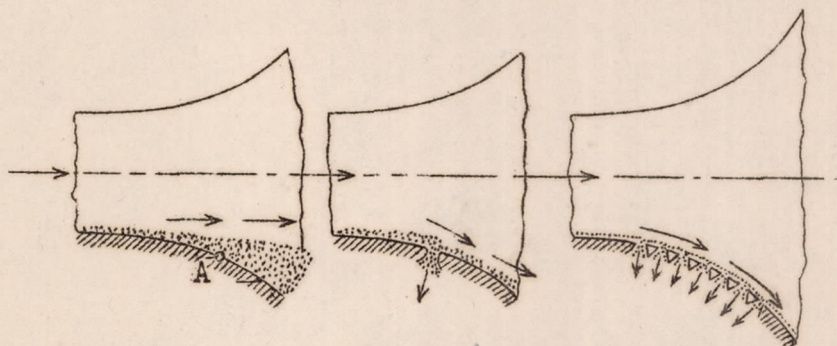


Fig.2 Diffuser without suction. Fig.3 Diffuser with suction. Fig.4 Diffuser with suction through successive slots.

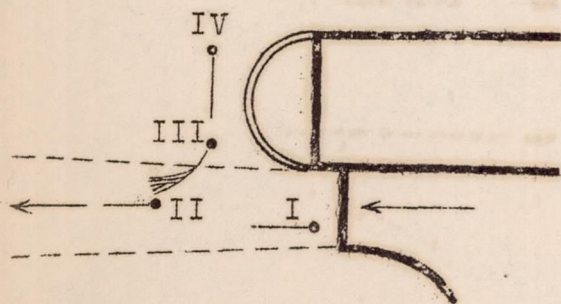


Fig.11 Interpretation of Fig.9.

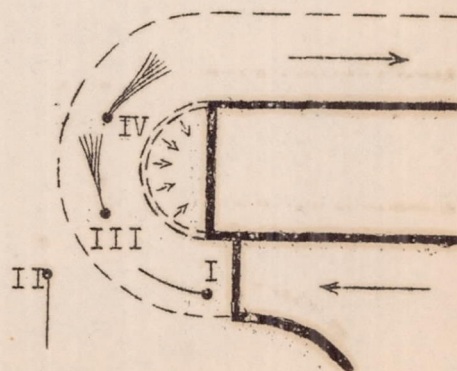


Fig.12 Interpretation of Fig.10.

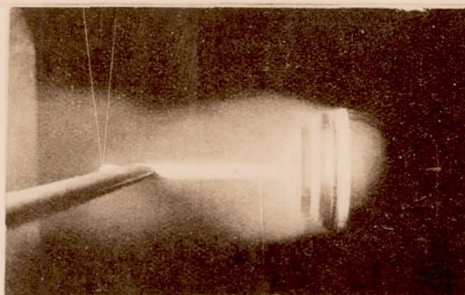


Fig. 5 Turbulent region without removal of boundary layer by suction.

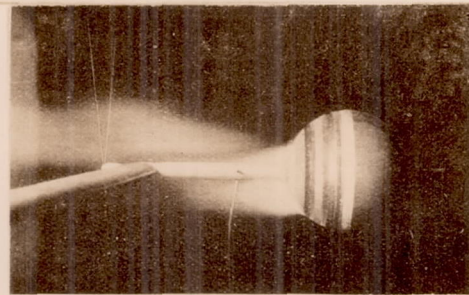


Fig. 6 Turbulent region with removal of boundary layer by suction.

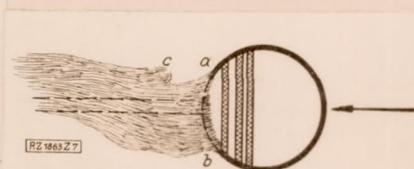


Fig. 7 Diagram for interpreting Fig. 6.

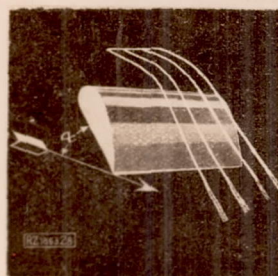


Fig. 8 Airfoil with boundary layer removed by suction from the upper side. Angle of attack 45° .

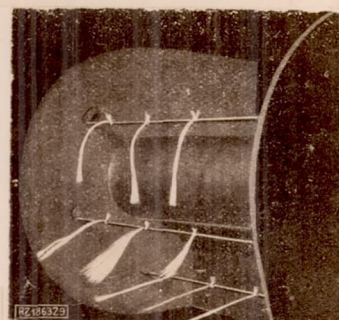


Fig. 9 Airflow undeflected.

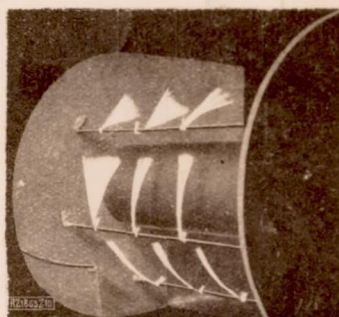


Fig. 10 Airflow made to conform by suction.



Fig. 18

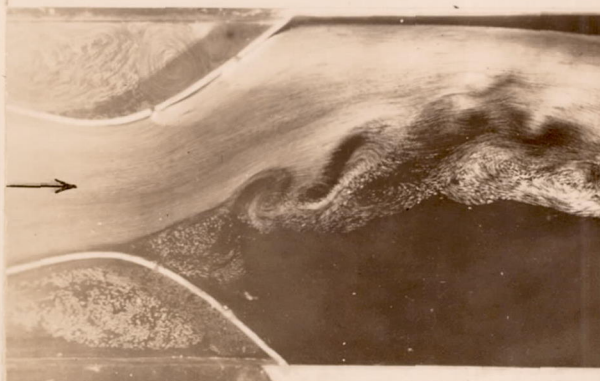


Fig. 19

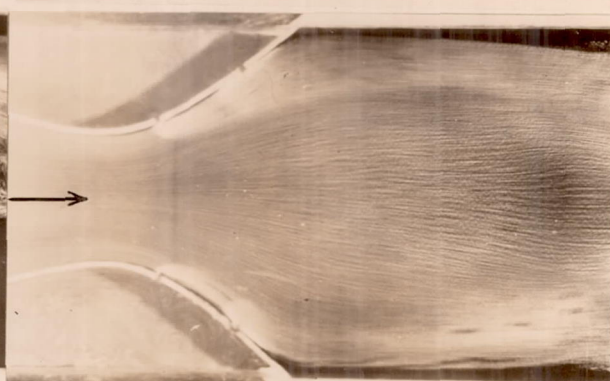


Fig. 20

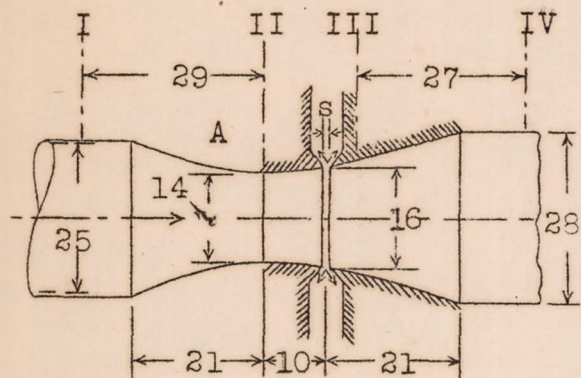


Fig.13 Diffuser, type A.

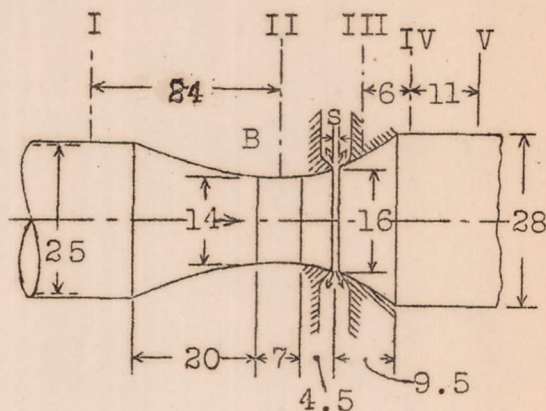


Fig.14 Diffuser, type B.

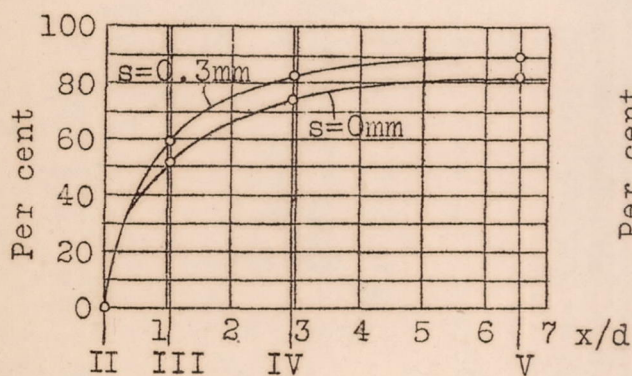


Fig.15 Pressure in diffuser A.

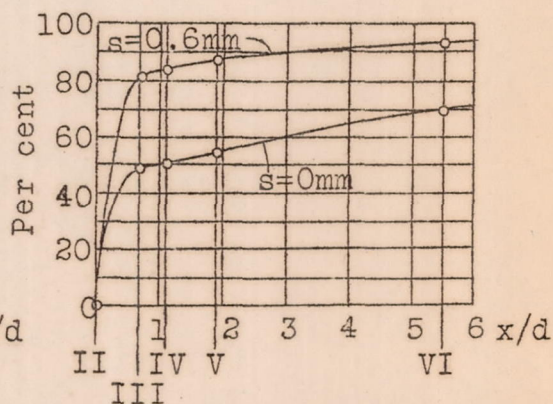


Fig.16 Pressure in diffuser B.

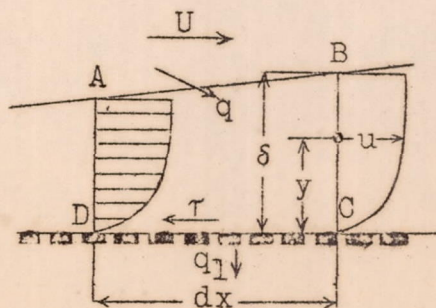


Fig.17 On theory of removal by suction.